Firms and Workers III

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Matching workers in firms

- In Lecture 9 and Lecture 10, we looked at models which assumed workers are heterogeneous in skills.
- How should these workers be matched in firms?
- Today, we will look at a model that tackles this question, producing many insights relevant to understanding firms in LMICs.
- The model is based on the idea of skill complementarities, exemplified in the story of the O-ring.

Matching workers in firms

- In Lecture 9 and Lecture 10, we looked at models which assumed workers are heterogeneous in skills.
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- The model is based on the idea of skill complementarities, exemplified in the story of the O-ring.

Challenger Space Shuttle

1986: Space shuttle Challenger explodes shortly after launch.

The immediate cause of the failure was the disintegration of an O-ring seal.

An O-ring is a rubber part of the kind used in many applications to provide a tight seal between metal parts.

Challenger Space Shuttle

The O-ring was not designed for cold temperatures; the material became brittle in the cold.

On the launch day, temperatures were far colder than those under which the shuttle had been tested for safety.

A failure of the O-rings led to a catastrophic and tragic failure in the mission.

The physicist Richard Feynman was part of the official commission charged with investigating the causes of the disaster. His demonstration of the issue in a public hearing is considered a masterpiece of science education...

O Ring video

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Kremer (1993)

Kremer (1993) built on this episode to theorize a production process in which the creation of a finished good requires the completion of a series of tasks.

Basic idea:

- A mistake in any one task can reduce the product's value.
- The tasks depend on one another and thus are complementary.
- An extreme version of this is the 'weakest link' problem: the strength of a chain is strength of its weakest link.
- 'Remember that the velocity of the guerrilla band on the march is equal to the velocity of its slowest man' Che Guevara, *Guerrilla Warfare*

Garment manufacturing in the Hawassa Industrial Park, Ethiopia



Complementarities in speed and quality of output



Model setup

Consider a production process that requires n separate tasks, where n is initially exogenous.

For simplicity, assume each task requires a single worker.

Different workers have different skill, denoted by their quality q.

Model setup

Worker quality q is defined by the average percentage of maximum value the product retains if the worker performs the task. This could mean:

- that the worker always produce goods worth q of the maximum value.
- that the worker completes the task perfectly q percent of the time.
- that the worker produces the good perfectly half the time and the other half the time produces goods that are worth $(2q - 1)$.

The meaning of q

For example, $q = 0.9$ could mean that a worker:

- always produces goods with 90% of the potential value
- produces goods perfectly 90% of the time and destroys them 10% of the time
- produces them perfectly 50% of the time and produces them with 80% of potential value the other 50% of the time

All of these will be equivalent.

Tasks are complementary

Different tasks are complementary in production.

When one task fails, it affects the marginal value of all other tasks.

- When one worker destroys the good, it destroys the (successful) product of other workers.
- Tasks are multiplicative, in this sense, rather than additive

The firm

Let B denote the output per worker of the firm with a single unit of capital if all tasks are performed perfectly.

Total output is thus nB.

Capital is denoted k and enters the production function in a Cobb-Douglas fashion.

All capital is the same (no differentiation by quality).

The firm

- Expected production is then:

$$E(y) = k^{\alpha} q_1 q_2 \dots q_n \; nB$$

or for notational simplicity:

$$E(y) = k^{\alpha} \left(\prod_{i=1}^{n} q_i \right) nB \tag{1}$$

- The complementarity here implies that quantity cannot substitute for quality: two poor workers do not replace one good worker.
- To see this consider what would happen if you replace a worker with $q = 1$ for two workers with $q = .5$. Or what happens if you get a worker with $q = 0$.

Firm's problem

- Suppose that in the short run, the firm has a fixed supply of capital, k^*.
- The firm is presumed to be risk neutral.
- Labour is supplied inelastically; i.e., workers face no labour-leisure trade-off.
- There is a distribution of worker quality given by $\phi(q)$.

Assortative matching of workers implied by profit maximization

- In this model, firms will always match the quality of their workers.
- Intuition:
  - Suppose you have two workers of quality q_1 and two workers of quality q_2.
  - Suppose $q_1 > q_2$.
  - One firm has hired a worker of quality 1, another has hired a worker of quality 2.
  - The former gains more by hiring the remaining quality 1 worker and hence will attract them by bidding their wage up.

Assortative matching of workers: efficiency

Note also that the output that you will get from matching workers with like quality will be greater than you get from mixing workers across quality.

- To see this, suppose that $q_1 - q_2 = a$, where $a > 0$. Then note that:

$$q_1^2 + q_2^2 = q_1(q_2 + a) + q_2(q_1 - a) = 2q_1q_2 + a(q_1 - q_2) > 2q_1q_2$$

$\rightarrow$ Production efficiency requires that workers within a firm have same quality.

The firm's problem

- More formally, we can derive the same intuition from the firm's optimization problem.
- Assume that individual firms are small and are price takers relative to the labour market.
- Each worker quality category will have its own wage; we call this $w(q)$.
- There is a rental rate for capital, denoted by r.
- The firm chooses capital and the skill of each worker (q_i) to maximise profits:

$$\max_{k,\{q_i\}} \; k^{\alpha} \left(\prod_{i=1}^{n} q_i \right) nB - \sum_{i=1}^{n} w(q_i) - rk$$

First-order conditions

- There is a first-order condition for each worker. This is given by:

$$\frac{dw(q_i)}{dq_i} = k^{\alpha} \left(\prod_{j \neq i} q_j \right) nB \tag{2}$$

- The left-hand side shows how much additional wage the firm will need to pay to upgrade the skills of the *i*th worker.
- The RHS shows $\frac{dy}{dq_i}$: the amount of additional output the firm gets from upgrading the worker's skills; the marginal product of skill for the *i*th worker.

$\rightarrow$ the output a firm gets by replacing one worker with a slightly higher skill worker, while holding constant the skill of all other workers, must equal the increase in the wage bill needed to hire the better worker.

Cross-partial

- Now consider how the marginal product of quality for a particular worker interacts with the quality of all the other workers:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}q_i \, \mathrm{d}\left(\prod_{j \neq i} q_j\right)} = k^{\alpha} nB \tag{3}$$

- This is surely positive...
- What this is telling us is that there is complementarity between the skills of an individual worker and the skills of the fellow workers with whom s/he is matched.

Assortative matching of workers implied by profit maximisation (again!)

- This implies that firms that employ workers of quality-level q gain the most and are prepared to bid the most to hire the marginal quality-q worker.
- In equilibrium, firms will employ identically-skilled workers for all tasks.
- Every worker employed by the firm will have the same q.
- This is a form of assortative matching.

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Competitive equilibrium in a simple O-ring model

- Consider a simple version of the O-ring model, with no capital in the production function.

$$y = q_1 \dots q_n nB = \prod_{i=1}^{n} q_i \, nB$$

- There are endlessly many potential firms, each with a production technology like this one.
- Workers are of type $q \in [0, 1]$.
- Assume that the total labor supply is given by N, uniformly distributed over skill levels $[0, 1]$.
- The measure of workers with skill less than q is given by qN for any value of q.

Characterising competitive equilibrium

- A competitive equilibrium is a specification of the wage schedule $w(q)$ for each type of labor q such that the quantity supplied of labor is equated with the quantity demanded, and further that each firm's profit is zero.
- For each firm faced with the wage schedule $w(q)$ the maximization problem is:

$$\max_{\{q_i\}} \prod_{i=1}^{n} q_i \, nB - \sum_{i=1}^{n} w(q_i)$$

This gives a first order condition for each task i:

$$w'(q_i) = \prod_{j \neq i} q_j \, nB \tag{1}$$

This is a necessary condition for the optimum.

As before, any equilibrium will have to show assortative matching: each firm only employs workers of one level of quality.

A competitive equilibrium

- Given assortative matching, FOC (1) becomes

$$w'(q) = q^{n-1} nB$$

- Profits have to be zero, so $q^n nB - nw(q) = 0$, Thus:

$$w(q) = q^n B$$

- Firm's profit-maximization condition is consistent with zero profits: the competitive equilibrium will hold.
- Note the shape of the wage schedule: skill premium rises sharply with the number of tasks!

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(1) small differences in skills have large output implications

Small differences in worker quality can make a big difference to aggregate productivity in complex production processes.

For instance, suppose that producing a car requires 20 tasks to produce a maximum value B.

Production quality differs across countries. Suppose the following are values for each country:

- $q = 0.99$ in Japan
- $q = 0.97$ in Vietnam

A numerical note

We can compute the value of production in Japan and Vietnam:

- In Japan:
$$y_J = Bk^{\alpha}(0.99)^{20}$$
- In Vietnam:
$$y_V = Bk^{\alpha}(0.97)^{20}$$

With equal value of capital, the ratio of production values is:

$$\frac{y_J}{y_V} = \frac{0.82}{0.54} \approx 1.5$$

This implies that, all else equal, the value of production in Japan is 1.5 times that in Vietnam.

(2) assortative matching: Finding workers with the right skills is crucial for firms

- It is optimal for firms to employ workers with the same level of skills for their various tasks.
- This requires them to be able to observe the skills that worker have.

- Can firms accurately infer the skills of prospective workers?

(3) Workers are paid more in high-skill firms

- Workers in high-skill firms are paid more, controlling for the level of their skill.
- This is because their productivity increases due to the high productivity of their co-workers.

$\rightarrow$ A janitor at Ethiopian airlines will be paid more than a janitor at a small restaurant even if the two have essentially the same skills.

(4) Firms will produce more complex products in richer countries

- Consider a simple extension: firms can choose n — think of this as choosing the complexity of their product.
- Assume that $B'(0) > 0$ and $B''(n) < 0$: output increases in tasks when n is small, but at a diminishing rate.
- Result: firms that choose more tasks will require a higher level of skill q (since mistakes become more costly when there are more tasks).
- Poorer countries have a less skilled workforce, i.e. it is hard to find high q workers for firms.
- Thus, firms in poorer countries may not find it optimal to produce complex products.

$\rightarrow$ Rich countries will produce more complex products than poor countries.

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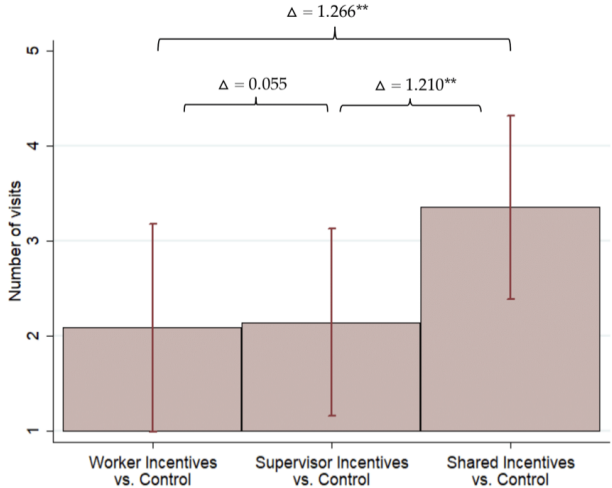
References

How large are complementarities in practice?

Deserranno et al. 2025 provide evidence on this question

- An experiment in the context of a large community health initiative in Sierra Leone;
- Teams composed of one supervisor and several and frontline workers;
- How should incentive be allocated in the team?
- Randomly vary (i) the introduction of a piece rate, and (ii) the share of the piece rate allocated to the worker (p)
- $p \in \{0, .5, 1\}$

Shared Incentives Maximize Number of Visits



Notes: Mean in the control group = 5.334.
*** p<0.01, ** p<0.05, * p<0.1

▸ Table

They rationalise these results with a model that features effort complementarities

A worker (supervisor) choose effort e_1 (e_2) to maximise their payoff:

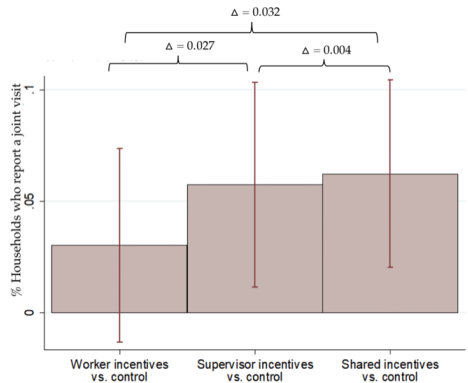
$$\pi_1 = (b_1 + \bar{q}(pm + s - \bar{k}) * y(e_1, e_2) - c(e_1) \tag{2}$$
$$\pi_2 = (b_2 + \bar{q}((1-p)m - zs) * y(e_1, e_2) - c(e_2) \tag{3}$$
$$\tag{4}$$

And the output function is given by

$$y = \alpha e_1 + \gamma e_1 e_2 \tag{5}$$

Supervisor Effort

- Supervisor effort does not ↑ monotonically with size of incentives received by supervisor ▸ Table



Notes: Mean in the control group = 0.164
*** p<0.01, ** p<0.05, * p<0.1

Estimate model parameters based on results on effort and output

Moments Fit

Back

Moments	Targeted Real	Simulated
Supervisor effort in worker incentives treatment	0.198	0.159
Supervisor effort in supervisor incentives treatment	0.225	0.278
Supervisor effort in shared incentives treatment	0.228	0.230
Supervisor effort in control group	0.164	0.111
Output in worker incentives treatment	10.551	10.908
Output in supervisor incentives treament	10.413	10.678
Output in shared incentives treament	11.827	11.255
Output in control group	7.256	7.174
Side payment in worker incentives treatment (1,000 SSL)		0.00
Side payment in supervisor incentives treatment (1,000 SSL)		0.000
Side payment in shared incentives treatment (1,000 SSL)		0.000
Value loss function	3.7	

Find evidence of a large complementarity

Table 7: Parameter Estimates

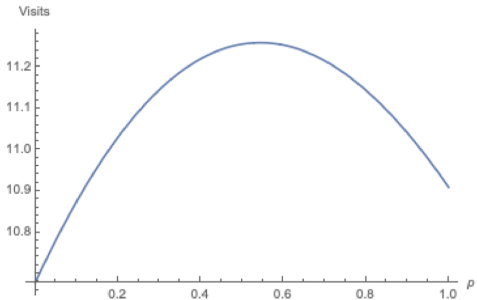
Complementarity γ	43.1
	(4.5)
Worker baseline incentive b_1	76.4
	(19.7)
Supervisor baseline incentive b_2	49.9
	(15.3)
α	6.7
	(0.7)
Calibrated friction z	3.61
Δ in marginal product of worker effort (shared incentive)	146%
Δ in marginal product of worker effort (control)	101%
Total worker cost of effort (control)	1555.4
Total supervisor cost of effort (control)	1022.3

Notes: The first panel of the table shows parameter estimates obtained using minimum distance estimation. We use eight empirical moments: supervisor effort in each one of the four treatments, and number of visits per month in each one of the four experimental groups. Supervisor effort is proxied by the proportion of households that receive a visit where the worker is accompanied by the supervisor. Costs are expressed in thousand SLL. Boostrapped standard errors are reported in parenthesis (we bootstrap the estimation 500 times and truncate the estimated coefficients at the 99^{th} percentile of the distribution). The second panel first shows the calibrated value of contractual frictions. Second, it shows some quantities implied by the parameter estimates. To calculate the change in the marginal product of worker effort we take the derivative of the production function with respect to worker effort (i) with $\gamma = 43.1$ and supervisor effort fixed at the level indicated in parenthesis, and (ii) with $\gamma = 0$. To calculate the total cost of an agent effort we multiply the unit cost of effort by the average effort exerted by the agent in the control group.

And infer that the optimal scheme is close to the shared incentive treatment

Counterfactual Policy: Optimal Incentive p^*

$p^* = 0.54$



Can firms observe the skills of prospective workers? Is matching frictionless?

Abebe et al. 2021 present experimental evidence on this question

Abebe et al. 2021 experimentally evaluate two programs:

1. a **job application workshop**
2. a **transport treatment**

The workshop is designed to make skills more observable. The transport treatment to make search less expensive.

The hypothesis is that treated subjects will search more *intensely* and *effectively*, leading to improved employment outcomes.

The **Job Application Workshop**

It involves two components:

1. **Orientation for effective job applications**:
   CVs, cover letters, interviews and use of the certificates
2. **Standardised tests**: cognitive, linguistic and mathematical ability and work sample test.

The cost of the intervention was 18.2 USD per person (excluding the cost of developing the tests).

The intervention was implemented by AA Commercial College.

The **Transport Treatment**

- They offer a monetary reimbursement, available at a central location, 3 times per week, for an average of 16 weeks.
- Calibrated to cover the cost of a single return trip to the centre.
  - Median = $ 1 , Max = $ 1.50, Min = $ 0.75.

The cost of the intervention was 19.8 USD per person.

Table: Employment outcomes

Outcome	2015			2018		
	Control mean (1)	Transport (2)	Workshop (3)	Control mean (4)	Transport (5)	Workshop (6)
Work	0.562	0.041 (0.029) [0.397]	0.021 (0.031) [0.666]	0.693	-0.063* (0.034) [0.305]	0.027 (0.031) [1.000]
Hours worked	26.18	0.268 (1.586) [0.946]	-0.254 (1.562) [1.000]	28.26	-2.636* (1.486) [0.305]	0.144 (1.404) [1.000]
Monthly earnings	1,145.0	4.8 (75.5) [0.946]	71.4 (83.9) [0.656]	1,533.7	27.1 (100.3) [0.715]	308.8** (123.4) [0.087]
Permanent job	0.171	0.029 (0.018) [0.392]	0.065*** (0.020) [0.008]	0.307	-0.038 (0.025) [0.305]	-0.011 (0.028) [1.000]
Formal job	0.224	0.054*** (0.019) [0.033]	0.051** (0.020) [0.029]	0.319	-0.006 (0.030) [0.715]	-0.006 (0.030) [1.000]
Job satisfaction	0.237	-0.001 (0.027) [0.946]	0.025 (0.027) [0.656]	0.574	-0.025 (0.036) [0.586]	0.069* (0.036) [0.159]

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