

Firms II

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Roadmap

Introduction and motivation

O-Ring Theory of Firms

A simplified model

Implications

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References

Big questions

- Why do we see rich countries specialising in the production of complicated products while poor countries produce simple commodities?
- Is this related to the fact that poor countries have many small firms?
- Is this related to the fact that there is a positive correlation between firm size and wages, within countries?

Challenger Space Shuttle

1986: Space shuttle Challenger explodes shortly after launch.

The immediate cause of the failure was the disintegration of an O-ring seal.

An O-ring is a rubber part of the kind used in many applications to provide a tight seal between metal parts.

Challenger Space Shuttle

The O-ring was not designed for cold temperatures; the material became brittle in the cold.

On the launch day, temperatures were far colder than those under which the shuttle had been tested for safety.

A failure of the O-rings led to a catastrophic and tragic failure in the mission.

The physicist Richard Feynman was part of the official commission charged with investigating the causes of the disaster. His demonstration of the issue in a public hearing is considered a masterpiece of science education...

[O Ring video](#)

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Kremer (1993)

Kremer (1993) built on this episode to theorize a production process in which the creation of a finished good requires the completion of a series of tasks.

Basic idea:

- A mistake in any one task can reduce the product's value.
- The tasks depend on one another and thus are complementary.
- An extreme version of this is the 'weakest link' problem: the strength of a chain is strength of its weakest link.
- 'Remember that the velocity of the guerrilla band on the march is equal to the velocity of its slowest man' Che Guevara, *Guerrilla Warfare*

Garment manufacturing in the Hawassa Industrial Park, Ethiopia



Complementarities in speed and quality of output



Comparison with previous models / lectures

Kremer (1993) builds on the work of others, including Lucas (1978), which build models of organizational hierarchy in which managerial skill enters the production function multiplicatively.

In those models that we have previously seen, agents with skill below some cutoff level become workers, and agents with skill above the cutoff level become managers. Higher skill managers supervise more employees.

Kremer (1993) differs in examining skill interaction among workers at the same level of hierarchy. In Lucas (1978), higher skills managers supervise more employees. What happens to high-skill workers in Kremer (1993)?

Model setup

Consider a production process that requires n separate tasks, where n is initially exogenous.

For simplicity, assume each task requires a single worker.

Different workers have different skill, denoted by their quality q .

Model setup

Worker quality q is defined by the average percentage of maximum value the product retains if the worker performs the task. This could mean:

- that the worker always produce goods worth q of the maximum value.
- that the worker completes the task perfectly q percent of the time.
- that the worker produces the good perfectly half the time and the other half the time produces goods that are worth $(2q - 1)$.

The meaning of q

For example, $q = 0.9$ could mean that a worker:

- always produces goods with 90% of the potential value
- produces goods perfectly 90% of the time and destroys them 10% of the time
- produces them perfectly 50% of the time and produces them with 80% of potential value the other 50% of the time

All of these will be equivalent.

Tasks are complementary

Different tasks are complementary in production.

When one task fails, it affects the marginal value of all other tasks.

- When one worker destroys the good, it destroys the (successful) product of other workers.
- Tasks are multiplicative, in this sense, rather than additive

The firm

Let B denote the output per worker of the firm with a single unit of capital if all tasks are performed perfectly.

Total output is thus nB .

Capital is denoted k and enters the production function in a Cobb-Douglas fashion.

All capital is the same (no differentiation by quality).

The firm

- Expected production is then:

$$E(y) = k^{\alpha} q_1 q_2 \dots q_n nB$$

or for notational simplicity:

$$E(y) = k^{\alpha} \left(\prod_{i=1}^n q_i \right) nB \quad (1)$$

- The complementarity here implies that quantity cannot substitute for quality: two poor workers do not replace one good worker.
- To see this consider what would happen if you replace a worker with $q = 1$ for two workers with $q = .5$. Or what happens if you get a worker with $q = 0$.

Firm's problem

- Suppose that in the short run, the firm has a fixed supply of capital, k^* .
- The firm is presumed to be risk neutral.
- Labour is supplied inelastically; i.e., workers face no labour-leisure trade-off.
- There is a distribution of worker quality given by $\phi(q)$.

Assortative matching of workers

- In this model, firms will always match the quality of their workers.
- Intuition:
 - Suppose you have two workers of quality q_1 and two workers of quality q_2 .
 - Suppose $q_1 > q_2$.
 - The output that you will get from matching workers with like quality will be greater than you get from mixing workers across quality.

Assortative matching of workers

- To see this, suppose that $q_1 - q_2 = a$, where $a > 0$. Then note that:

$$q_1^2 + q_2^2 = q_1(q_2 + a) + q_2(q_1 - a) = 2q_1q_2 + a(q_1 - q_2) > 2q_1q_2$$

- 1 Production efficiency requires that workers within a firm have same quality.
- 2 A firm that matches quality q will be willing to pay higher wages to attract q workers from less-well-paid jobs in firms that do not match quality.

The firm's problem

- More formally, we can derive the same intuition from the firm's optimization problem.
- Assume that individual firms are small and are price takers relative to the labour market.
- Each worker quality category will have its own wage; we call this $w(q)$.
- There is a rental rate for capital, denoted by r .
- The firm chooses capital and the skill of each worker (q_i) to maximise profits:

$$\max_{k, \{q_i\}} k^\alpha \left(\prod_{i=1}^n q_i \right) nB - \sum_{i=1}^n w(q_i) - rk$$

First-order conditions

- There is a first-order condition for each worker. This is given by:

$$\frac{dw(q_i)}{dq_i} = k^\alpha \left(\prod_{j \neq i} q_j \right) nB \quad (2)$$

- The left-hand side shows how much additional wage the firm will need to pay to upgrade the skills of the i th worker.
 - The RHS shows $\frac{dy}{dq_i}$: the amount of additional output the firm gets from upgrading the worker's skills; the marginal product of skill for the i th worker.
- the output a firm gets by replacing one worker with a slightly higher skill worker, while holding constant the skill of all other workers, must equal the increase in the wage bill needed to hire the better worker.

Cross-partial

- Now consider how the marginal product of quality for a particular worker interacts with the quality of all the other workers:

$$\frac{d^2 y}{dq_i d\left(\prod_{j \neq i} q_j\right)} = k^\alpha n B \quad (3)$$

- This is surely positive...
- What this is telling us is that there is complementarity between the skills of an individual worker and the skills of the fellow workers with whom s/he is matched.
- A worker has a higher marginal product when matched with good fellow workers.

Sorting

- This directly implies that firms don't want a single worker with skills below the average.
- In equilibrium, firms will employ identically-skilled workers for all tasks.
- Every worker employed by the firm will have the same q .
- This is a form of assortative matching.

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Competitive equilibrium in a simple O-ring model

- Consider a simple version of the O-ring model, with no capital in the production function.

$$y = q_1 \dots q_n nB = \prod_{i=1}^n q_i nB$$

- There are endlessly many potential firms, each with a production technology like this one.
- Workers are of type $q \in [0, 1]$.
- Assume that the total labor supply is given by N , uniformly distributed over skill levels $[0, 1]$.
- The measure of workers with skill less than q is given by qN for any value of q .

Characterizing competitive equilibrium

- A competitive equilibrium is a specification of the wage schedule $w(q)$ for each type of labor q such that the quantity supplied of labor is equated with the quantity demanded, and further that each firm's profit is zero.
- For each firm faced with the wage schedule $w(q)$ the maximization problem is:

$$\max_{\{q_i\}} \prod_{i=1}^n q_i nB - \sum_{i=1}^n w(q_i)$$

This gives a first order condition for each task i :

$$w'(q_i) = \prod_{j \neq i} q_j nB \quad (1)$$

This is a necessary condition for the optimum.

As before, any equilibrium will have to show assortative matching: each firm only employs workers of one level of quality.

A competitive equilibrium

- Given assortative matching, FOC (1) becomes

$$w'(q) = q^{n-1} nB$$

- Profits have to be zero, so $q^n nB - nw(q) = 0$, Thus:

$$w(q) = q^n B$$

- Firm's profit-maximization condition is consistent with zero profits: the competitive equilibrium will hold.
- Note the shape of the wage schedule: skill premium rises sharply with the number of tasks!

Applications to the real world

- Speaks to the large differences in productivity between rich and poor countries.
- O-ring production function creates a mechanism in which small differences in worker skill create large differences in productivity and wages.
- In equilibrium, more capital is matched with higher-skill workers.
- Firms in the same economy may hire different qualities of workers and produce different qualities of goods.
- The model predicts a positive correlation within enterprises among the wages of workers performing different jobs.
- Complex production processes will be very costly in economies with relatively few skilled workers.
- If it is costly for firms to screen workers for quality, there can be big costs from poor matching.

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(1) assortative matching + workers are paid more in high-skill firms

- Firms tend to employ workers with similar skills for their various tasks.
 - In other words, we should expect to find that the IT people in a high-powered and high-productivity organization will earn more than similarly qualified IT people in other organisations.
 - Even the cleaning staff is likely to earn more in a high-powered organization, even though their tasks are the same that they would perform in a low-productivity firm.
- In this model, due of the nature of production and matching, workers performing the same task earn higher wages in a high-skill firm than in a low-skill firm.

(2) small differences in skills have large output implications

Small differences in worker quality can make a big difference to aggregate productivity in complex production processes.

For instance, suppose that producing a car requires 20 tasks to produce a maximum value B .

Production quality differs across countries. Suppose the following are values for each country:

- $q = 0.99$ in Japan
- $q = 0.97$ in Vietnam

A numerical note

We can compute the value of production in Japan and Vietnam:

- In Japan:

$$y_J = Bk^\alpha(0.99)^{20}$$

- In Vietnam:

$$y_V = Bk^\alpha(0.97)^{20}$$

With equal value of capital, the ratio of production values is:

$$\frac{y_J}{y_V} = \frac{0.82}{0.54} \approx 1.5$$

This implies that, all else equal, the value of production in Japan is 1.5 times that in Vietnam.

(3) Finding workers with the right skills is crucial for firms

- A firm with a vacancy will not just want to simply hire the first worker they find.
- It will be valuable for them to find the worker with the *right* level of skills.
- This requires them to be able to observe the skills that worker have.
- Can firms accurately infer the skills of prospective workers?

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(4) Firms will be larger and produce more complex products in richer countries

- Suppose firms can choose n (tasks, but also number of workers per firm).
 - Richer countries, with a highly skilled workforce, will have a higher optimal n .
- Firms will be larger, all else equal, in rich countries.
- Rich countries will produce more complex products than poor countries.

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Abebe et al. 2021 present experimental evidence on this question

Abebe et al. 2021 experimentally evaluate two programs:

1. a **job application workshop**
2. a **transport treatment**

The workshop is designed to make skills more observable. The transport treatment to make search less expensive.

The hypothesis is that treated subjects will search more *intensely* and *effectively*, leading to improved employment outcomes.

The **Job Application Workshop**

It involves two components:

1. **Orientation for effective job applications:**
CVs, cover letters, interviews and use of the certificates
2. **Standardised tests:** cognitive, linguistic and mathematical ability and work sample test.

The cost of the intervention was 18.2 USD per person (excluding the cost of developing the tests).

The intervention was implemented by AA Commercial College.

The Transport Treatment

- They offer a monetary reimbursement, available at a central location, 3 times per week, for an average of 16 weeks.
- Calibrated to cover the cost of a single return trip to the centre.
 - Median = \$ 1 , Max = \$ 1.50, Min = \$ 0.75.

The cost of the intervention was 19.8 USD per person.

They randomize at the level of geographical clusters

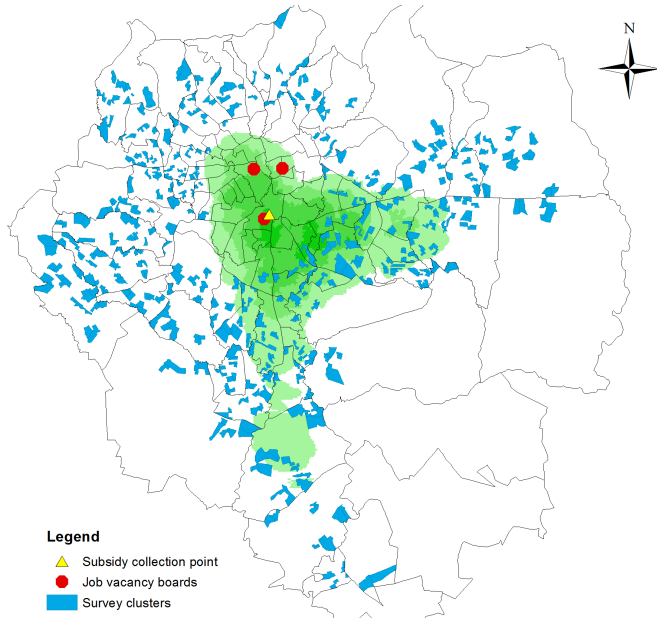


Table: Employment outcomes

Outcome	2015			2018		
	Control mean (1)	Transport (2)	Workshop (3)	Control mean (4)	Transport (5)	Workshop (6)
Work	0.562	0.041 (0.029) [0.397]	0.021 (0.031) [0.666]	0.693	-0.063* (0.034) [0.305]	0.027 (0.031) [1.000]
Hours worked	26.18	0.268 (1.586) [0.946]	-0.254 (1.562) [1.000]	28.26	-2.636* (1.486) [0.305]	0.144 (1.404) [1.000]
Monthly earnings	1,145.0	4.8 (75.5) [0.946]	71.4 (83.9) [0.656]	1,533.7	27.1 (100.3) [0.715]	308.8** (123.4) [0.087]
Permanent job	0.171	0.029 (0.018) [0.392]	0.065*** (0.020) [0.008]	0.307	-0.038 (0.025) [0.305]	-0.011 (0.028) [1.000]
Formal job	0.224	0.054*** (0.019) [0.033]	0.051** (0.020) [0.029]	0.319	-0.006 (0.030) [0.715]	-0.006 (0.030) [1.000]
Job satisfaction	0.237	-0.001 (0.027) [0.946]	0.025 (0.027) [0.656]	0.574	-0.025 (0.036) [0.586]	0.069* (0.036) [0.159]

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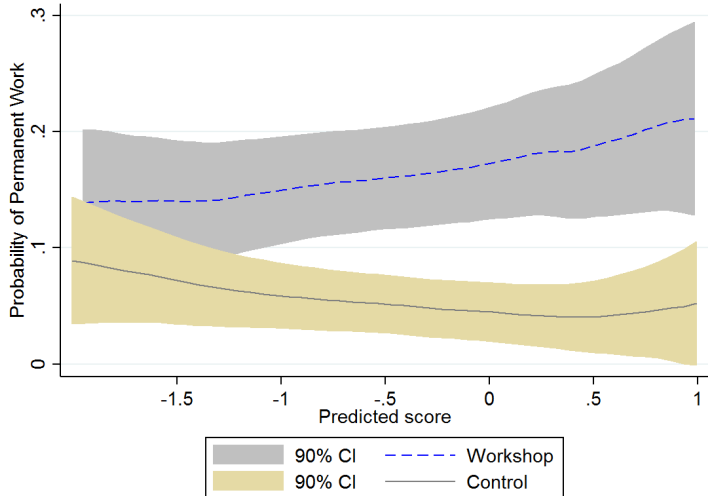
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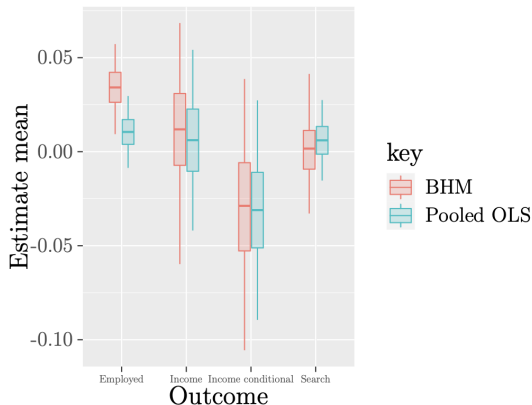
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The workshop increases the returns to observable skills



Meta-analysis of signalling interventions by Kreft corroborates the findings of the workshop intervention

Impacts on employment



Mean estimated treatment effect (thick midline), 50 percent interval (box) and 97 percent interval (thin line).

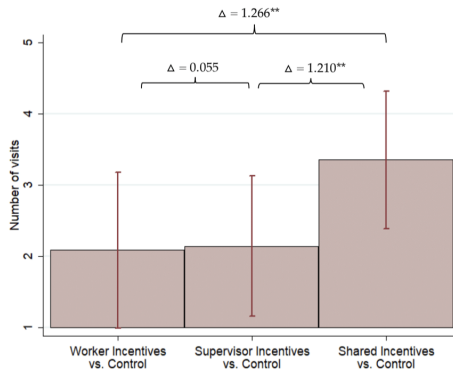
Estimates report average percentage point change in outcome as a result of treatment.

Estimates produced using the logistic model for binary outcomes and individual-level Rubin model for continuous outcomes.

How large are complementarities in practice? Evidence from Deserranno et al. 2025

Impacts on employment

Shared Incentives Maximize Number of Visits



Notes: Mean in the control group = 5.334.

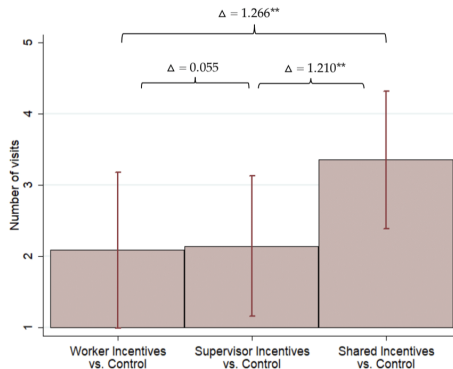
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► Table

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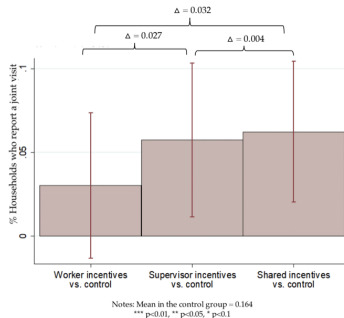
► Table

How large are complementarities in practice? Evidence from Deserranno et al. 2025

Impacts on employment

Supervisor Effort

- Supervisor effort does not $\uparrow$ monotonically with size of incentives received by supervisor [Table](#)



How large are complementarities in practice? Evidence from Deserranno et al. 2025

Impacts on employment

Moments Fit [▶ Back](#)

Moments	Targeted Real	Simulated
Supervisor effort in worker incentives treatment	0.198	0.159
Supervisor effort in supervisor incentives treatment	0.225	0.278
Supervisor effort in shared incentives treatment	0.228	0.230
Supervisor effort in control group	0.164	0.111
Output in worker incentives treatment	10.551	10.908
Output in supervisor incentives treatment	10.413	10.678
Output in shared incentives treatment	11.827	11.255
Output in control group	7.256	7.174
Side payment in worker incentives treatment (1,000 SSL)		0.00
Side payment in supervisor incentives treatment (1,000 SSL)		0.000
Side payment in shared incentives treatment (1,000 SSL)		0.000
Value loss function	3.7	

How large are complementarities in practice? Evidence from Deserranno et al. 2025

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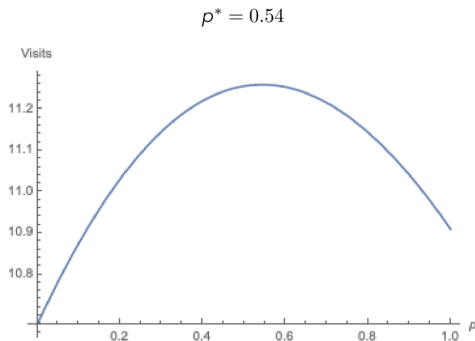
Parameters Estimates [Back](#)

	(1)
Complementarity γ	7.9
Contractual friction z	3.0
Worker unit cost of effort c_1	145.3
Supervisor unit cost of effort c_2	1611.1
Worker baseline incentive b_1	57.8
Supervisor baseline incentive b_2	19.0
α	5.1
Δ in marginal product of worker effort (shared incentive)	36 %
Δ in marginal product of worker effort (no incentive)	17 %
Total worker cost of effort (no incentive)	173.6
Total supervisor cost of effort (no incentive)	178.9

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Counterfactual Policy: Optimal Incentive p^*



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